Example Let $\{X_\alpha\}_{\alpha\in A}$ be a collection of topological spaces. Let $X = \prod_{\alpha\in A} X_\alpha$ be the product. Define $\pi_\alpha : X \to X_\alpha$ by $\pi_\alpha(x) = x_\alpha$. Then π_α is continuous.

Proof We say that a topology $\mathcal{T}$ on X is the weakest topology making π_α continuous if:

1) π_α is continuous w.r.t. $\mathcal{T}$
2) If $\mathcal{T}'$ is another topology such that π_α is continuous w.r.t. $\mathcal{T}'$, then $\mathcal{T} \subseteq \mathcal{T}'$.

Let $\mathcal{T}$ be the weakest topology making π_α continuous. Let $\mathcal{B} = \{\pi_\alpha^{-1}(U_\alpha) : \alpha \in A, U_\alpha \text{ open in } X_\alpha\}$. Then $\mathcal{B}$ is a basis for $\mathcal{T}$.

Example Let $X = \mathbb{R}^\omega$. Let $\mathcal{T}_{box}$ be the box topology. Let $\mathcal{T}_{prod}$ be the product topology. Then $\mathcal{T}_{prod} \subseteq \mathcal{T}_{box}$.

Proof Let $\mathcal{B}_{box}$ be the box basis. Let $\mathcal{B}_{prod}$ be the product basis. Let $U \in \mathcal{B}_{box}$. Let $x \in U$. Let $\varepsilon_i > 0$ be such that $(x_i - \varepsilon_i, x_i + \varepsilon_i) \subset U_i$. Let $\varepsilon = \min(\varepsilon_1, \varepsilon_2, \ldots)$. Then $(x_1 - \varepsilon, x_1 + \varepsilon) \times (x_2 - \varepsilon, x_2 + \varepsilon) \times \cdots \subset U$. Hence $\mathcal{B}_{box} \subseteq \mathcal{T}_{prod}$. Hence $\mathcal{T}_{box} \subseteq \mathcal{T}_{prod}$.

Example Let $X = \mathbb{R}^\omega$. Let $\mathcal{T}_{box}$ be the box topology. Let $\mathcal{T}_{prod}$ be the product topology. Let $\mathcal{T}_{uniform}$ be the uniform topology. Let $\mathcal{T}_{uniform}$ be the uniform topology induced by $\bar{d}(x,y) = \sup_i \min(|x_i - y_i|, 1)$. Then $\mathcal{T}_{box} \supseteq \mathcal{T}_{uniform} \supseteq \mathcal{T}_{prod}$.

Proof Let $\mathcal{B}_{box}$ be the box basis. Let $\mathcal{B}_{prod}$ be the product basis. Let $\mathcal{B}_{uniform}$ be the uniform basis. Let $U \in \mathcal{B}_{uniform}$. Let $x \in U$. Let $\varepsilon > 0$ be such that $B_{\bar{d}}(x, \varepsilon) \subset U$. Let $\varepsilon_i = \min(\varepsilon, 1)$. Then $(x_1 - \varepsilon_1, x_1 + \varepsilon_1) \times (x_2 - \varepsilon_2, x_2 + \varepsilon_2) \times \cdots \subset U$. Hence $\mathcal{B}_{box} \supseteq \mathcal{B}_{uniform}$. Hence $\mathcal{T}_{box} \supseteq \mathcal{T}_{uniform